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Quark–Gluon Plasma in Equilibrium State

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The thermodynamic potential of quark–gluon plasma (QGP) in equilibrium state is calculated by finite temperature QCD. The pressure correction of QGP and the critical temperature correction of deconfinement phase transition of hadron are discussed.

KEY WORDS: equilibrium; quark–gluon plasma; pressure; phase transition.

1. INTRODUCTION

With indirect evidence of the Top quark found, all six quarks have been verified to be existent. But up to now it is found that all the hadron states that can be observed in isolation are color singlets and a single quark in isolation is not observed, which suggests quarks are confined inside a hadron. A useful phenomenological description of quarks in hadrons is provided by the bag model. While there are many different versions of the model (Bardeen *et al.*, 1975; Brown and Rho, 1979; Lee, 1987), the MIT bag model contains the essential characteristics of the phenomenology of quark confinement (Chodos *et al.*, 1974). In the model quarks are treated as massless particles inside a bag of finite dimension, and are infinitely massive outside the bag. Confinement in the model is the result of the balance of the bag pressure B (Wong, 1994), which is directed inward, and the stress arising from the kinetic energy of the quarks and the gluons inside the bag. Due to a hadron being a colorless particle, the quarks and the gluons inside the hadron form a quark–gluon plasma (QGP) and the QGP is thought to exist in an equilibrium state in general.

Lattice gauge theory predicts that a deconfinement phase transition of the hadron occurs at high temperature and high density and a QGP can be produced (Engels, 1997; Fucito *et al.*, 1984). The QGP is believed to have existed in the early stage of the universe, $\sim 10^{-4}$ s after the big bang. The ultra-relativistic heavy-ion collisions offer the unique opportunity to study the production of QGP in the laboratory. The space-time evolution of the QGP during an ultra-relativistic

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heavy-ion collision might proceed through different stages such as: 1) the pre-equilibrium, 2) the equilibrium, and 3) the hadronization stage (Ruppert *et al.*, 2001). Hence in the paper we want to investigate the properties of QGP in equilibrium state. First a thermodynamical potential of the QGP is calculated, then the pressure of the QGP and the temperature of deconfinement phase transition of hadron are discussed.

2. CALCULATION OF THERMODYNAMIC POTENTIAL OF QGP

QGP in equilibrium state is described by finite temperature QCD. According to the theory of finite temperature field (Kapusta, 1989), a thermodynamic potential of the QGP can be calculated by a perturbative method, a zero approximation of the thermodynamic potential is the contribution of ideal gas and the other order corrections are the contributions of the vacuum graphs when the interaction is considered. The zero approximation (density of thermodynamical potential) is (Wong, 1994)

$$\Omega^{(0)} = \frac{37\pi^2 T^3}{90}. \tag{1}$$

Now we calculate the first-order correction of the thermodynamical potential which are the contributions of two-loop vacuum graphs shown in Fig. 1(a)–(d). Due to high-loop calculations, the real-time formalism of temperature green function is used because the difficult infinity sum will be met if the imaginary-time formalism is used. The thermopropagators of quark, gluon (Feynman gauge), and ghost fields respectively are the following (Landsman and Van Weert, 1987),

$$iS_\beta^{ij} = \delta^{ij} k \left(\frac{i}{k^2 + i\eta} + 2\pi \underline{\delta}_k \right), \tag{2}$$

where $i, j = 1, 2, 3$ are color indices; $\quad k = \gamma^\mu k_\mu, \quad \underline{\delta}_k = -\frac{\delta(k^2)}{e^{\beta|k_0|}+1}$.

$$iD_{\mu\nu}^{ab} = -\delta^{ab} g_{\mu\nu} \left(\frac{i}{k^2 + i\eta} + 2\pi \delta_k \right), \tag{3}$$

where $\delta_k = \frac{\delta(k^2)}{e^{\beta|k_0|}-1}$.

$$iG_{\mu\nu}^{ab} = -\delta^{ab} \left(\frac{i}{k^2 + i\eta} + 2\pi \delta_k \right). \tag{4}$$

The calculations corresponding to four Feynman graphs in Fig. 1 are

$$\begin{aligned} I_a = -\frac{N_f g^2}{2} \int \frac{d^D p d^D k}{(2\pi)^{2D}} \left(\frac{i}{p^2 + i\eta} + 2\pi \underline{\delta}_p \right) \left(\frac{i}{(p+k)^2 + i\eta} + 2\pi \underline{\delta}_{p+k} \right) \\ \times \left(\frac{i}{k^2 + i\eta} + 2\pi \delta_k \right) N_a, \end{aligned} \tag{5}$$

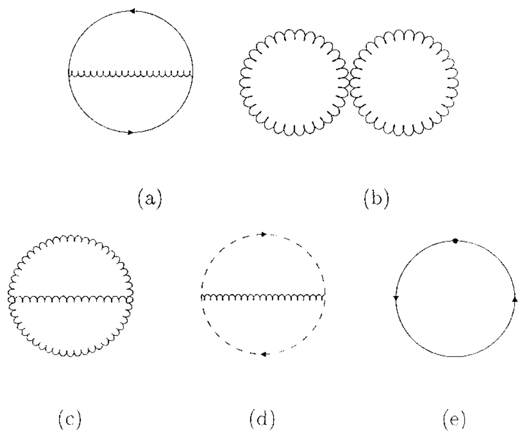

Fig. 1. Two-loop vacuum graphs and counterterm graphs.

where N_f comes from flavors of quark.

$$
\begin{aligned}
N_a &= Tr[\gamma^{\mu} T^{a} (\not{p} + m) \gamma^{\nu} T^{b} (\not{p} + \not{k} + m)] \delta^{ab} g_{\mu\nu} \\
&= -4(N^2 - 1)(p^2 + p \cdot k - 2m^2).
\end{aligned} \tag{6}
$$

$$
I_b = \frac{1}{8}(-ig^2) \int \frac{d^D p d^D k}{(2\pi)^{2D}} \left(\frac{i}{p^2 + i\eta} + 2\pi \delta_p \right) \left(\frac{i}{k^2 + i\eta} + 2\pi \delta_k \right) N_b, \tag{7}
$$

$$
N_b = (-1)^2 W^{abcd}_{\mu\nu\rho\sigma} \delta^{ad} \delta^{bc} g^{\mu\sigma} g^{\nu\rho} = 24N(N^2 - 1), \tag{8}
$$

where $W^{abcd}_{\mu\nu\rho\sigma}$ is vertex function of four-gluon interaction.

$$
\begin{aligned}
I_c = -\frac{g^2}{12} \int \frac{d^D p d^D k}{(2\pi)^{2D}} &\left(\frac{i}{p^2 + i\eta} + 2\pi \delta_p \right) \left(\frac{i}{(p+k)^2 + i\eta} + 2\pi \delta_{p+k} \right) \\
&\times \left(\frac{i}{k^2 + i\eta} + 2\pi \delta_k \right) N_c,
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
N_c &= (-1)^3 f^{abc} V_{\mu\nu\lambda}(-p-k,\, p,\, k) f^{a'c'b'} V_{\alpha\beta\rho} \delta^{aa'} \delta^{bb'} \delta^{cc'} g^{\mu\alpha} g^{\lambda\beta} g_{\nu\rho} \\
&= 18N(N^2 - 1)(p^2 + p \cdot k + k^2),
\end{aligned} \tag{10}
$$

where $V_{\mu\nu\lambda}(p, q, k)$ is vertex function of three-gluon interaction.

$$
\begin{aligned}
I_d = -\frac{g^2}{2} \int \frac{d^D p d^D k}{(2\pi)^{2D}} &\left(\frac{i}{p^2 + i\eta} + 2\pi \delta_p \right) \left(\frac{i}{(p+k)^2 + i\eta} + 2\pi \delta_{p+k} \right) \\
&\times \left(\frac{i}{k^2 + i\eta} + 2\pi \delta_k \right) N_d,
\end{aligned} \tag{11}
$$

$$\begin{aligned} N_d &= (-1)^2[f^{abc}p_\mu f^{b'a'c'}(p+k)_\nu]\delta^{cc'}g^{\mu\nu}\delta^{aa'}\delta^{bb'} \\ &= -N(N^2-1)(p^2+p\cdot k). \end{aligned} \tag{12}$$

In above equations, we can separate the thermodynamical potential into two parts, the zero temperature part and finite temperature part. Here we only consider the finite temperature part because the zero temperature part is not related to temperature and only gives out a constant after renormalization. There are ultraviolet divergences in Fig. 1 (a), which may be removed by considering a counterterm. The counterterm of Fig. 1(a) is Fig. 1(e), where dot denotes renormalization constant that came from one-loop self-energy graph of quark. Minimum Scheme being taken account of, the result of calculation of Fig. 1(e) is

$$\begin{aligned} I_e &= \frac{N_f g^2}{16\pi^2\epsilon}\delta^{ij}\int\frac{d^D p}{(2\pi)^{D-1}}Tr(4m-\not{p})(\not{p}+m)\delta^{ji}\underline{\delta}_p \\ &= -12m^2N_f(N^2-1)\frac{g^2}{32\pi^4\epsilon}\int_0^\infty dp\frac{p^2}{\sqrt{p^2+m^2}}\,\frac{1}{e^{\beta\sqrt{p^2+m^2}}+1}. \end{aligned} \tag{13}$$

In Figs. 1(b)–1(d), due to gluon field and ghost field being the massless fields, some integrals in these graphs may become the following form,

$$\int\frac{d^D p}{(2\pi)^D}\frac{1}{p^\alpha}, \qquad \alpha > 0. \tag{14}$$

According to prescription of dimensional regularization, the integral equals to zero (Muta, 1987). Therefore, there is no ultraviolet divergence in these graphs.

The calculation results of the finite temperature parts of Figs. 1(b)–1(d) respectively are

$$\begin{aligned} I_a = 2(N^2-1)N_f g^2\Bigg[&3(m^2)^{1+\epsilon/2}\frac{i\Gamma(2-D/2)}{(4\pi)^{D/2}2\pi^2}\int_0^\infty dp\frac{p^2}{\sqrt{p^2+m^2}}\,\frac{1}{e^{\beta\sqrt{p^2+m^2}}+1} \\ &-i\frac{1}{8\pi^4}\int_0^\infty dp\frac{p^2}{\sqrt{p^2+m^2}}\,\frac{1}{e^{\beta\sqrt{p^2+m^2}}+1}\int_0^\infty dk\frac{k^2}{\sqrt{k^2+m^2}}\,\frac{1}{e^{\beta\sqrt{k^2+m^2}}+1} \\ &+i\frac{m^2}{8\pi^4}\int_0^\infty dpdk\,\frac{1}{e^{\beta\sqrt{p^2+m^2}}+1}\,\frac{1}{e^{\beta\sqrt{k^2+m^2}}+1}\,\frac{p}{\sqrt{p^2+m^2}}\,\frac{k}{\sqrt{k^2+m^2}} \\ &\times\ln\left|\frac{(m^2-pk)^2-(k^2+m^2)(k^2+m^2)}{(m^2+pk)^2-(p^2+m^2)(k^2+m^2)}\right| \\ &-i\frac{1}{2\pi^2}\left(\frac{1}{12\beta^2}\right)\int_0^\infty dp\frac{p^2}{\sqrt{p^2+m^2}}\,\frac{1}{e^{\beta\sqrt{p^2+m^2}}+1}\Bigg], \end{aligned} \tag{15}$$

$$I_b = -3ig^2N(N^2-1)\left(\frac{1}{12\beta^2}\right)^2, \tag{16}$$

$$I_c = \frac{9ig^2}{4} N(N^2 - 1) \left(\frac{1}{12\beta^2} \right)^2 , \tag{17}$$

$$I_d = -\frac{ig^2}{4} N(N^2 - 1) \left(\frac{1}{12\beta^2} \right)^2 . \tag{18}$$

Summing them, the first-order correction of the thermodynamic potential is obtained,

$$\begin{aligned}
\Omega_1 &= -i\beta(I_a + \ldots + I_e) \\
&= g^2\beta_2(N^2-1)N_f \left[\frac{3m^2}{32\pi^4} \left(-\gamma_E + \ln\frac{4\pi}{m^2} + \frac{3}{2} \right) \int_0^\infty dp \frac{p^2}{\sqrt{p^2+m^2}} \frac{1}{e^{\beta\sqrt{p^2+m^2}}+1} \right. \\
&\quad - \frac{1}{8\pi^4} \int_0^\infty dp \frac{p^2}{\sqrt{p^2+m^2}} \frac{1}{e^{\beta\sqrt{p^2+m^2}}+1} \int_0^\infty dk \frac{k^2}{\sqrt{k^2+m^2}} \frac{1}{e^{\beta\sqrt{k^2+m^2}}+1} \\
&\quad + \frac{m^2}{8\pi^4} \int_0^\infty dp\, dk \frac{1}{e^{\beta\sqrt{p^2+m^2}}+1} \frac{1}{e^{\beta\sqrt{k^2+m^2}}+1} \frac{p}{\sqrt{p^2+m^2}} \frac{k}{\sqrt{k^2+m^2}} \\
&\quad \times \ln\left| \frac{(m^2-pk)^2-(k^2+m^2)(k^2+m^2)}{(m^2+pk)^2-(p^2+m^2)(k^2+m^2)} \right| \\
&\quad \left. - \frac{1}{2\pi^2} \left(\frac{1}{12\beta^2} \right) \int_0^\infty dp \frac{p^2}{\sqrt{p^2+m^2}} \frac{1}{e^{\beta\sqrt{p^2+m^2}}+1} \right] - g^2\frac{N(N^2-1)}{144} T^3.
\end{aligned} \tag{19}$$

3. PRESSURE OF QUARK–GLUON PLASMA

Next we discuss the pressure of QGP in equilibrium state. According to thermodynamics theory, the relation between thermodynamic potential and pressure is

$$P = T\Omega. \tag{20}$$

When the QGP is the ideal gas, the zero approximation of pressure is (Wong, 1994)

$$P^{(0)} = T\Omega^{(0)} = 37\frac{\pi^2}{90}T^4. \tag{21}$$

In the calculations the mass of quark is ignored in general because the QGP is produced at high temperature and the momentum of quark is very large. Inserting Eq. (19) into Eq. (20) and taking $m = 0$, we obtain the first-order correction of the pressure,

$$P^{(1)} = -g^2 N_f \frac{5(N^2-1)}{4\times 144} T^4 - g^2 \frac{N(N^2-1)}{144} T^4. \tag{22}$$

The sign of $P^{(1)}$ in Eq. (22) is negative, which means that the pressure is decreased after considering the interactions among particles. At finite temperature, the coupling constant is related to temperature and needs renormalization. In one-loop approximation (Fujimoto, 1987) it is

$$g_r^2 = \frac{g^2}{1 + (2Ng^2/4\pi)[8\pi^3T^2/9\sqrt{3}M + 2\pi^2T/3M + A]}, \tag{23}$$

where A is a constant not to be related to temperature, M is a parameter of renormalization group equation. In Eq. (23) there only remains the term of temperature square in denominator because the temperature of the system is very high and we have

$$P^{(1)} = \frac{9\sqrt{3}M}{4N\pi^2}\left[N_f\frac{5(N^2-1)}{4\times 144} + \frac{N(N^2-1)}{144}\right]T^2. \tag{24}$$

Comparing Eq. (24) with Eq. (21), we know $P^{(1)}$ is very less than $P^{(0)}$ because $P^{(1)}$ is the order of temperature square and $P^{(0)}$ is the order of T^4. Combining the two terms, we obtain the pressure

$$P = P^{(0)} + p^{(1)} = 37\frac{\pi^2}{90}T^4 - \frac{9\sqrt{3}M}{4N\pi^2}\left[N_f\frac{5(N^2-1)}{4\times 144} + \frac{N(N^2-1)}{144}\right]T^2. \tag{25}$$

4. CORRECTION OF CRITICAL TEMPERATURE OF DECONFINEMENT PHASE TRANSITION OF HADRON

In the MIT bag model, the quarks are treated as massless particles inside a bag of finite dimension, and the perturbation QCD is valid in the region because the non-Abelian gauge theories have the property known as asymptotic freedom (Quigg, 1983). When the quarks and the gluons inside the bag are idealized to be noninteracting and massless and there is no net baryon number, the partial pressure arising from the quarks, antiquarks, and the gluons is

$$P^{(0)} = T\Omega^{(0)} = 37\frac{\pi^2}{90}T^4. \tag{26}$$

The bag pressure B is 206 MeV (Wong, 1994). Let $P^{(0)} = B$, then the critical temperature of hadron deconfinement phase transition may be estimated,

$$T_c \sim 144\,\text{MeV}. \tag{27}$$

After the pressure being corrected, in Eq. (25) taking $N = 3$, $N_f = 2$, the pressure is

$$P = 37\frac{\pi^2}{90}T^4 - g^2\frac{11}{36}T^4. \tag{28}$$

Let the pressure $= B$, then the critical temperature of deconfinement phase transition may be calculated. Although it is related to coupling constant, we can be sure that the pressure would be decreased after the correction and the critical temperature would be increased. Owing to a quark being in asymptotic freedom in the region, the coupling constant is less, the increase of the temperature is not big. Several special values are given in the following, while $g^2 = 0.1$, $T_c \sim 144.5$ MeV; $g^2 = 0.5$, $T_c \sim 145.3$ MeV; and $g^2 = 1$, $T_c \sim 146.7$ MeV.

5. SUMMARY

In the paper the thermodynamic potential of QGP in equilibrium state is calculated by finite temperature QCD. Using the relation between thermodynamic potential and pressure, the correction of the QGP pressure is discussed. The result indicates that the pressure is decreased due to the interactions among particles. In the MIT bag model, the critical temperature of hadron deconfinement phase transition is estimated. The temperature is increased with the decrease of the pressure arising from the kinetic energy of quarks and gluons owing to the interactions among particles. But the quarks are in weak coupling region inside the MIT bag and the coupling constant is less, the correction of critical temperature of deconfinement phase transition still is small.

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